On rank 3 groups and partial linear spaces

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A partial linear space is an incidence structure consisting of points and lines such that every line contains at least 3 points and every pair of points is in at most a line. I will assume partial linear spaces to be finite, not graphs nor linear spaces. Going all the way back to my PhD dissertation, I have been interested in partial linear spaces with varying degrees of symmetry, called k-ultrahomogeneity. The weakest level of symmetry among these is when $k = 2$: when the automorphism group is transitive on the ordered pairs of collinear points and on the ordered pairs of non-collinear points. This is equivalent to the automorphism group having rank 3.

Primitive rank 3 groups are classified. In 2005 I classified partial linear spaces admitting a primitive almost simple rank 3 group, and in 2008 the ones with a primitive rank 3 group of grid type. The case of primitive affine rank 3 groups was much harder to tackle, but was finally done in 2021, with Joanna Fawcett, John Bamberg and Cheryl Praeger (except for a few 'hopeless' cases.) Imprimitive rank 3 groups in general have not been classified, but in 2006, with Jonathan Hall, we managed to do the easiest case, when all lines have size 3. Recent work provided classification of imprimitive rank 3 groups with some extra assumptions. Rank 3 quasiprimitive groups were classified in 2011 (AD, Michael Giudici, Cai Heng Li, Geoffrey Pearce, Cheryl Praeger) and rank 3 innately transitive in 2023 (Anton Baykalov, AD and Cheryl Praeger), and rank 3 quasiprimitive groups in 2025 (Hong Yi Huang, Cai Heng Li, Yan Zhou Zhu, Hanyue Yi). With Anton Baykalov and Cheryl Praeger, we have now classified the partial linear spaces with such groups, finding some nice infinite families and a small number of sporadic examples.